# THE GROWTH OF A DEPOSITED LAYER ON A COLD SURFACE<sup>†</sup>

# PAUL A. LIBBY; and SHUN CHEN§

Department of Aerospace Engineering and Applied Mechanics, Polytechnic Institute of Brooklyn, Brooklyn, New York, U.S.A.

(Received 24 July 1964 and in revised form 19 October 1964)

Abstract—An approximate analysis of the growth of, and the temperature within, a deposit which may occur in a cold surface in a gas stream is developed with the convective heat transfer to the surface of the deposit taken into account. Simple "short time" and "long time" solutions are obtained. A numerical example which may be of interest in connection with cryogenic surfaces in a hypersonic wind tunnel is presented.

## NOMENCLATURE

- A,  $\theta(\pi' + L')H_{\alpha}^3$ , parameter;
- B,  $2\{(\theta-1) + H_{\infty} + 6(L'+H')\theta\}H_{\infty},$ parameter;
- b, constant defined by equation (6);
- c. constant defined by equation (16);
- $g_w$ , stagnation enthalpy ratio,  $h_w/h_s$ , e;
- $H^*$ , arbitrary constant;
- *H*, perturbed dependent variable, cf. equation (19);
- H,  $hq_{c, 0}/k_a T_R$ , non-dimensional thickness of the deposited layer;
- $H_{\infty}$ , limiting value of H at  $\tau \rightarrow \infty$ ;
- *h*, thickness of deposited layer;
- $h_i$ , specific enthalpy of species i;
- $h_w$ , stagnation enthalpy at the surface  $T = T_f$ ;
- $h_{s, e}$ , stagnation enthalpy of the free gas stream;
- $h_{\infty}$ , thickness of deposited layer at  $t \to \infty$ ;
- h, rate of growth of the despoted layer;
- $k_d$ , thermal conductivity of deposit;
- L, latent heat of condensation from the gas phase to the solid phase;
- L',  $(a_d \rho_d/k_d)(L/T_f)$ , non-dimensional latent heat:

† This research was sponsored by the Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, under Contract AF 33(657)-8286.

<sup>‡</sup> Professor of Aerospace Engineering; presently Professor, Department of Aerospace and Mechanical Engineering Sciences, University of California San Diego.

§ Research Fellow.

- Q,  $q_w h/k_d$ , non-dimensional heat-transfer rate;
- $q_c$ , convective heat-transfer rate;
- $q_{c,0}$ , convective heat-transfer rate with no deposition;
- $q_w$ , total heat-transfer rate;
- t, time;
- T, temperature;
- $T_R$ , temperature of the plate;
- $T_f$ , temperature of exposed surface of the deposited layer;
- *u*, velocity component in *x* direction;
- v, velocity component in z direction;
- x, coordinate in streamwise direction;
- z, coordinate perpendicular to x direction;
- ad, thermal diffusivity;
- $\beta$ , coefficient defined by equation (23);
- $\eta, z/h;$
- $\theta$ ,  $T_f/T_R$ , non-dimensional temperature;
- $\lambda$ , exponent in equation (21);
- $\mu$ , coefficient of viscosity;
- $\pi$ ,  $\partial q_c/\partial h$ , mass-heat-transfer parameter;
- $\pi'$ ,  $(a_d/k_d)(\pi/T_f)$ , parameter;
- $\rho$ , density;
- $\tau$ ,  $tq_{c,0}^2 a_d/k_d^2 T_k^2$ , non-dimensional time;
- $\tau^*$ , arbitrary non-dimensional time.

# Subscripts

- d, condition in deposited layer;
- i, species i;
- e, free stream condition;
- g, gas phase condition.

## I. INTRODUCTION

THERE appears to be a variety of applications in boundary-layer control, cryogenic engineering, low density wind tunnel operation, and geophysics involving fluid flow over surfaces with temperatures sufficiently low so that deposition of the fluid, or of components thereof in case of mixtures, occurs on the surface. In particular, it has been suggested by Cresci and Visich [1] that the performance of a supersonic compressor for use in a drive system for a hypersonic wind tunnel may be improved if boundary-layer control as obtained by gas solidification on cryogenically cooled blades is employed. The occurrence of frost on cryo-surfaces in air streams has been studied by Smith et al. [2] and appears to be a frequently encountered problem in cryogenic engineering (cf. [3]). Chuan and Karamcheti [4] have described the operation of a low density, two phase wind tunnel which involves condensation of the working fluid downstream of the test section on a cryo-surface; further studies on the deposition phenomena connected with this application were carried out by them and are presented in [5] and [6]. Finally, the formation of ice on the surface of lakes and oceans has apparently been studied by geophysicists (cf. [7]).

None of the cited references which actually discuss the heat conduction within the deposited layer as it grows in thickness with time appear to include the effect of convective heating which generally accompanies the phase change from liquid or gas to solid. Accordingly, such solutions apply only for either "short time" or "thin laver" deposits. On the contrary, in several of the applications cited above, the "long time" behavior, which as will be seen below, is determined by the convective heating, is of importance. Moreover, quantitative estimates of the meaning of either "short time" or "thin layers" for a particular problem can only be made if there is available a more complete analysis; this does not appear to have been carried out to date. Accordingly, it is the purpose of this report to present an analysis of the growth of a deposited layer with convective heat transfer considered and to indicate on the basis thereof the essential influence of such heat transfer.

It will be assumed that the heat conduction

within the deposit may be treated as onedimensional in terms of a coordinate normal to a cold solid surface.<sup>†</sup> According to this approximation the spatial distribution of the thickness of the deposit at a given instant of time can be constructed provided the spatial distribution of convective heating is known. The moving boundary representing the surface of the deposit introduces considerable complexity into the analysis; therefore, it is found necessary to develop an approximate solution based on the integral method of Goodman [8].

As the analysis is presented below, it will be observed that several of the techniques and assumptions employed have been previously used in connection with the sublimation of a surface exposed, for example to a hypersonic flow. In this connection the work of Sutton [9] and of Economos [10] are especially appropriate. It is noted, however, that the deposition and sublimation problems are essentially different; the former will be shown below to lead to an asymptotic thickness as time increases whereas the latter is known to lead to a steady rate of recession of the surface as time increases.

## **II. ANALYSIS**

Consider an element of a deposit at a particular instant of time and the coordinate system shown in Fig. 1. According to the one-dimensional assumption cited above and to the further assumption that the thermal diffusivity of the deposit  $a_d$  is constant,<sup>‡</sup> the temperature history is described by the equation

$$T_t = a_d T_{zz} \tag{1}$$

† This assumption could be put in quantitative form by considering the more complete heat conduction equations, e.g. for two-dimensional, unsteady systems, and by introducing two length scales, one corresponding to the coordinate along the solid surface, L, the second normal thereto,  $\delta$ . Denote  $\tau = \delta/L \ll 1$  and consider a series expansion of the temperature and convective heating in powers of  $\tau$ . The first-order terms will correspond to those considered here. Singular behavior may occur at leading edges. The first author is indebted for this point of view to Dr. Frank Lane of the General Applied Science Laboratories.

An anonymous reviewer has called attention to possible errors between experiment and the predictions of the present analysis because of this assumption; there is no difficulty connected with removal of this  $a_d = \text{constant restriction}$ .



FIG. 1. Schematic representation of the conduction model.

with the conditions

$$T(0, t) = T_R$$
$$T(h, t) = T_t$$

and

$$k_d T_z(h; t) = q_c + \rho_d L h. \tag{2}$$

Equation (2) which presents the boundary condition of the moving surface perhaps requires some discussion.

# The boundary condition at the moving surface

Shown in Fig. 2 are the factors contributing to mass and energy conservation at the moving surface when a multi-component gas is depositing thereon and when heat conduction occurs from the gas to the deposit. The energy balance when N' species are depositing would be, at z = h(t),

$$(kT_z)_g - (kT_z)_d - h \left[ \sum_{i=1}^{N'} \rho_i h_i(T_f) \right]_d - \left[ \sum_{i=1}^{N'} \rho_i v_i h_i(T_f) \right]_g = 0 \qquad (3)$$

while the mass balance would be

$$-\left[\sum_{i=1}^{N'}\rho_i v_i\right]_g - \rho_a \dot{h} = 0.$$
 (4)

Now let  $q_c \equiv (kT_z)_g$  and take N' = 1, i.e. assume only one component of the gas is depositing; then equations (3) and (4) yield

$$(kT_z)_d = q_c + \rho_d h\{[h_1(T_f)]_g - [h_1(T_f)]_d\}$$

which is equation (2).

In previous analyses (cf. [5]–[7]) the contribution  $q_c$  to the boundary condition at z = h(t) is not included. It is perhaps worth noting that there would appear to be cases in which  $q_c < 0$ ; e.g. suppose a gas in a non-equilibrium state flows at low speed over a solid surface with a temperature  $T_e$  in the external flow,  $T_e < T_f$ . Then, if condensation occurs on the solid surface h > 0, but  $q_c < 0$ . In the present report only the case  $q_c > 0$ , which seems from the above cited references to be the more prevalent, will be considered explicity, although the analysis below is clearly applicable to the case  $q_c < 0$ and although quite different growth of the deposit would be expected.

The solution of the problem posed by equation (1) subject to its boundary conditions does not appear to be possible by analytic means for  $q_c \neq 0$ . However, for "short time" it would be expected on physical ground that  $\rho_d Lh \ge q_c$  and thus that for such times the classical solution prevails (cf. [5]-[7]); the thickness of the deposit is given by this solution as

$$h = 2b(a_d t)^{1/2} \tag{5}$$

where the constant b is given by the transcendental equation

$$be^{b^2} \operatorname{erf}(b) = (k_d/a_d \rho_d L \pi^{1/2})(T_f - T_R)$$
 (6)

$$\begin{pmatrix} (k T_z)_g - \left[ \sum_{i} \rho_i v_i h_i (T_r) \right]_g \\ (k T_z)_d \end{pmatrix}^{-1} h \left[ \sum \rho_i h_i (T_r) \right]_d$$

# ENERGY BALANCE



MASS BALANCE

FIG. 2. Energy and mass balance at the moving surface.

This analytic solution could be used to provide initial data for finite difference calculation starting at t > 0. However, here an approximate solution based on the integral method due to Goodman [8] is sought. Proceed as follows: Integrate equation (1) with respect to z from zero to the variable limit h; there is obtained after introduction of the boundary conditions, the equation

$$(h \int_{0}^{1} T \, \mathrm{d}\eta)_{t} - T_{f} h = \alpha_{d} \left[ (q_{w}/k_{d}) - (T_{z})_{0} \right] \quad (7)$$

where  $\eta = z/h$ . Now assume a temperature profile

$$T = T_f - (T_f - T_R) (1 - 2\eta + \eta^2) - Q(\eta - \eta^2)$$
(8)

Where  $Q = q_w h/k_d$ ; equation (8) satisfies all the boundary conditions at  $\eta = 0,1$  for any Q. Substitution of equation (8) into equation (7) leads to

$$2hh [(T_f - T_R) + (Q/2)] + h^2 \dot{Q} = -12a_d [Q - (T_f - T_R)] \quad (9)$$

which is an equation in two unknowns, h and Q. One more equation comes from equation (2) with the definition of Q introduced; thus

$$Q = (h/k_d)(q_c + h\rho_d L) \tag{10}$$

Differentiation of equation (10) and elimination of Q and Q by use of equation (10) and of the result of that differentiation yields a single second-order equation in h. In carrying cut the differentiation it is assumed that  $q_c$  depends on h, i.e. that the convective heat transfer is influenced by the rate of deposition, so that a term  $(\partial q_c/\partial h)$  arises; in particular the following linear form will be assumed:

$$q_c = q_{c, 0} + \pi h \tag{11}$$

where  $q_{c,0}$  is the convective heat transfer to a surface with  $T = T_f$  and with no deposition and where  $\pi = (\partial q_c / \partial h)$  is a proportionality factor accounting for the influence of the deposition on the convective heat transfer. Representations of the type implied by equation (11) have been found to be useful in studies of the sublimation of surfaces in hypersonic flows where h < 0 and where mass transfer into the gas stream leads to a reduction, i.e. to the so called "blockage effect", of the convective heating (cf. [9] and [10]). The general analysis of the local history of a deposit can be carried forward without explicit representations for  $q_{c,0}$  and  $\pi$ ; for a steady gas stream these parameters are constants for each element of deposit but, in general, vary with distance along the cold surface. It will be recognized that an accurate representation for  $q_{c, 0}$  and  $\pi$  will be difficult to provide even for relatively simple geometric and fluid mechanical situations; even for laminar flow there must be considered a non-similar boundary layer with time as a parameter and with the mass transfer determined simultaneously with the present analysis of the growth of the deposit applied locally in a spatial sense.

The final equation for h may be put in a convenient form if the following non-dimensional, dependent and independent variables are introduced:

$$\begin{array}{l} H \equiv hq_{c,\ 0}/k_d T_R \\ \pi = tq_{c,\ 0}^2 \alpha_d/k_d^2 T_r^2 \end{array}$$
 (12)

and if the following parameters, which relate to the properties of the gas and of the deposit and are independent of the flow properties as contained in  $q_{c, 0}$ , are introduced:

$$\begin{cases} \pi' \equiv (\alpha_d/k_d)(\pi/T_f) \\ L' \equiv (\alpha_d \rho_d/k_d)(L/T_f) \\ \theta \equiv T_f/T_R \end{cases}$$
 (13)

There results

$$\begin{bmatrix} \theta(\pi' + L') \end{bmatrix} H^3 (d^2 H/d\tau^2) + \{2(\theta - 1) \\ + 2[1 + \pi'\theta(dH/d\tau)]H + 2L'\theta H(dH/d\tau) \\ + 12L'\theta + 12\pi'\theta\} H(dH/d\tau) + 12H = \\ 12(\theta - 1), \quad (14)$$

Note that a solution to equation (14) permits Q to be computed as a function of  $\tau$  from equation (10) with equation (11) substituted and with non-dimensionalization carried out. Thus the temperature profiles can be estimated for various times.

## Initial conditions

One of the initial conditions  $H \sim h = 0$  at

t = 0 is evident on physical grounds; the second must be deduced from the behavior of equation (14). Clearly, a singularity occurs at t = 0; the solution for "short times" given by equation (5) and (6) suggests the assumption

$$H = c\tau^{1/2} \tag{15}$$

where c is a constant to be determined. Substituting into equation (14) leads to the result that

$$c = \left[\frac{2(\theta - 1) + 12L'\theta + 12\pi'\theta}{(\pi' + L')\theta}\right]^{1/2} \\ \left[\left\{1 + \frac{12(\pi' + L')(\theta - 1)\theta}{[(\theta - 1) + 6L'\theta + 6\pi'\theta]^2}2\right\}^{1/2} - 1\right]^{1/2}$$
(16)

as  $\tau \to 0$ . Thus, equations (15) and (16) provide starting values for H and  $dH/d\tau$  for  $\tau > 0$ ,  $c\tau^{1/2} \ll \theta - 1$ . Indeed, these equations provide an approximate solution which may be compared to the exact "short time" solution given by equations (5) and (6); for this comparison reintroduce h, t and L into equations (15) and (16) and set  $\pi' = 0$  as suggested by the neglect of  $q_c$ in this solution; then

$$h = c(a_d t)^{1/2} \tag{17}$$

and comparison of equations (5) and (7) indicates that c should be considered an approximation to 2b. The two constants considered as functions of  $(k_d/a_d\rho_d L)(T_f - T_R)$  are compared in Fig. 3 over a range of this parameter believed



FIG. 3. Comparison of approximate and exact "shorttime" solutions.

to be of interest; equation (8), specialized to  $\pi' = 0$ , may be more convenient for determining the "short-time" solution than equation (6). Note also that the above inequality provides an estimate of the time duration for which the "short-time" solution provides an adequate description of the phenomena.

## Asymptotic behavior

An examination of equation (14) in the limit as  $\tau \to \infty$  under the assumption that  $dH/d\tau$ ,  $d^2H/d\tau^2 \to 0$  indicates that H approaches a constant value  $H_{\infty}$  given by

$$H_{\infty} = \theta - 1. \tag{18}$$

This leads to the physically recognizable result that

$$h_{\infty} = k_d (T_f - T_R)/q_{c, 0}$$

i.e. the deposit will approach a limiting thickness dictated by the convective heating without mass transfer. Note that  $h_{\infty} \equiv 0$ , as  $q_{c, 0} \rightarrow \infty$ , as occurs, e.g. at the leading edge of a plate.

The actual approach to this asymptotic value  $H_{\infty}$  can be obtained by linearization about  $H_{\infty}$ ; i.e. in the usual fashion, let

$$H \simeq H_{\infty} + H, H \ll H_{\infty}.$$
 (19)

Then, equation (14) leads to

$$A(\mathrm{d}^{2}\tilde{H}/\mathrm{d}\tau^{2}) + B(\mathrm{d}\tilde{H}/\mathrm{d}\tau) + 12\tilde{H} = 0 \quad (20)$$

where

$$A = \theta(\pi' + L')H_{\infty}^{3}$$
  
$$B = 2\{(\theta - 1) + H_{\infty} + 6(L' + \pi')\theta\}H_{\infty}^{3}.$$

The solution for  $\overline{H}$  under the assumption that there occurs only one physically acceptable root to the secular equation<sup> $\dagger$ </sup> is

$$\dot{H} = H^* \exp\left[-\lambda\tau\right] \tag{21}$$

where  $\lambda > 0$ , and given by

$$A\lambda^2 - B\lambda + 12 = 0 \tag{22}$$

and where  $H^*$  is an arbitrary constant selected so that H has a desired value at  $\tau = \tau^*$ , e.g. the value given by a numerical solution of equation (14) carried out to  $\tau = \tau^*$ .

† Clearly, for some values of the parameter  $\pi'$ , L',  $\theta$  two positive values of  $\lambda$  might be obtained but it would be expected that one could be ruled out on physical grounds. Indeed this was found to be the case in the numerical example considered below.

#### The parameter $\pi$

As mentioned previously the parameter  $\pi$  accounts in a rough manner for the effect of mass transfer on convective heat transfer. Empirically, it might be assumed that the correlation found useful for subliming bodies in hypersonic flows would apply with a change of sign; if so, then, e.g. from [9]-[12], it is reasonable to let

$$\pi = \beta \rho_d \left( h_{s, e} - h_w \right) \tag{23}$$

where the value of  $\beta$  is on the order of unity and depends on whether the flow is laminar or turbulent and on the ratio of the molecular weight of the deposited specie and of the gas mixture in the external flow; and where  $h_w$ must be estimated from the temperature  $T_f$  and the concentration at the surface.

## The spatial distribution of deposit

If it is assumed that the parameters  $\pi'$ , L' and  $\theta$  are constants for a given situation involving deposition, then the solution  $H = H(\tau)$  given by the above analysis can be applied to determine for a given time t the thickness of the deposit h = h(x) provided the variation  $q_{c,0} =$  $q_{c,0}(x)$  is known. For example, suppose a surface such as a flat plate is being considered; for this  $q_{c, 0} \sim x^{-1/2}$  and since  $h_{\infty} \sim q_{c, 0}^{-1}$ , the deposit will have zero thickness for all times at the leading edge and will grow with x at a particular time in a manner so as to correspond to the solution  $H = H(\tau)$  over the entire range of  $\tau \ge 0$ . On the contrary, for a stagnation point where  $q_{c,0}$  is constant with x, h = h(t) will be independent of x.

## **III. NUMERICAL EXAMPLE**

To illustrate application of the above analysis the deposition on a flat plate of nitrogen under conditions of possible interest in a hypersonic wind tunnel has been considered.<sup>†</sup> Equation (14) was rearranged as two first-order equations and integrated by standard methods on the P. I. B-IBM 7040 computer. The physical and thermo. dynamic data for nitrogen were obtained from [13] and were

$$L = 60 \text{ cal/g} a_d = 6.29 \times 10^{-4} \text{ cm}^2/\text{s} \rho_d = 1 \text{ g/cm}^3 k_d = 2.455 \times 10^{-4} \text{ cal/s cm }^\circ\text{K}$$
(24)

which leads to

$$L' = 4.05.$$
 (25)

The value of  $T_f$  was assumed to be 38°K and the value of  $\theta$  to be 2. An estimation of  $\beta$  in equation (23) was made on the basis of laminar boundary-layer theory for similar flow with  $h_{s, e} = 280$  cal/g and  $g_w = 0.1$ ; this leads to

$$\pi' = 11.9.$$
 (26)

Finally, to obtain estimates of the spatial distribution of the deposited layer at various times it was assumed that

$$q_{c,0} = 0.47 \ \rho_e u_e h_{s,e}, (1 - g_w) (2 \rho_e u_e x / \mu_e)^{-1/2}$$
(27)

which is the usual equation for laminar, flat plate heat transfer. If this expression for  $q_{c,0}$  is substituted into the definitions of H and  $\tau$ [cf. equation (12)], it is clear that a solution  $H = H(\tau)$  can be interpreted to yield the spatial thickness distribution in the form  $(\rho_e u_e h/\mu_e)$  $(\mu_e h_{s, e}/k_a T_r)$  as a function of  $(\rho_e u_e x/\mu_e)$  with the time in the form  $(\mu_e h_{s, e}/k_a T_R)^2 (\rho_e u_e/\mu_e)^2 (\alpha_a t)$  as a parameter. Thus in addition to the physical parameters stated above and required for solution of the growth of the deposit it is necessary to know the unit Reynolds number  $\rho_e u_e/\mu_e$  of the gas stream and the parameter  $(\mu_e h_{s, e}/k_a T_r)$ as well. With these selected h = h(x, t) can be readily computed.

It is perhaps of interest to note that in the neighborhood of the leading edge the "long time" solution provides the spatial distribution for all times sufficiently long so that  $\tau > \tau^*$ . Thus at a given time *t*, the thickness is distributed with x such that  $h - h_{\infty} \sim x^{1/2} \exp[-\lambda/x]$  where  $\lambda$  is a positive constant. On the contrary for far downstream distances such that  $\tau$ 

<sup>&</sup>lt;sup>†</sup> After completion of this study McDermott (Fourth International Symposium on Rarefied Gas Dynamics, July 1964, Toronto) presented some experimental results concerning condensation on the cryogenically cooled walls of a hypersonic nozzle. It would be of interest to compare the results of the present analysis with these experiments.



FIG. 5. Distribution of deposit with time as parameter-flat plate.

satisfies the equalities for the short time solution the thickness is independent of x.

The numerical solution for  $H = H(\tau)$  and the two limiting solution for "short-time" and "long-time" are shown in Fig. 4. Figure 5 presents the distribution of deposit with time as a parameter. Note the use of the "long time" solution near the leading edge as discussed above.

It will be noted that the main features of these two presentations of the one numerical example, i.e. Figs. 4 and 5, are in accord with the previous qualitative considerations regarding the two limiting solutions and regarding the spatial distribution on a flat plate. It is reiterated that the solution of  $H = H(\tau)$  in Fig. 4 can be directly applied to obtain the spatial and temporal history of deposits in other flow geometries. In particular for a stagnation point wherein  $q_{c,0}$  is constant with  $x, h \sim H$  and  $t \sim \tau$  so that with proper scaling Fig. 4 will give the time history of a deposit at a stagnation point.

## REFERENCES

- 1. R. J. CRESCI and M. VISICH, JR., Use of a supersonic compressor for a high enthalpy test facility. Proceedings of the Third Hypervelocity Techniques Symposium, Denver, Colorado, 17-18 March, 1964 (also to the available as an ARL report).
- 2. R. V. SMITH, D. K. EDMONDS, E. G. F. BRENTARI and R. J. RICHARDS, Analysis of the frost phenomena on a cryo-surface. Preprint of Paper No. B-7 prepared for

presentation at the 1963 Cryogenic Engineering Conference at the University of Colorado, 19–21 August 1963 (May be obtained from the Cryogenic Data Center, National Bureau of Standards Boulder, Colorado.)

- 3. D. B. CHELTON, B. W. BIRMINGHAM and J. W. DEAN, Solid formation in flowing cryogenic fluids *Advanc*. *Cryogenic Engng* 8, 311–314 (1963).
- 4. R. L. CHUAN and K. KARAMCHETI, Principle and application of the two-phase wind-tunnel system. University of Southern California Engineering Center, Report 42-101 (1955).
- R. L. CHUAN, A study of the condensation of nitrogen below the triple point. University of Southern California Engineering Center, Report No. 56-201 (1957).
- 6. K. KARAMCHETI, A note on the thermal conductivity of solid nitrogen and the direct condensation of nitrogen gas into a solid. University of Southern California Engineering Center Report No. 56-206 (1959).
- 7. L. D. INGERSOLL, O. J. ZOBEL and A. C. INGERSOLL,

Heat conduction with engineering geological and other applications, pp. 190-199. The University of Wisconsin Press (1954).

- T. R. GOODMAN, The heat-balance integral and its application to problems involving a change of phase, *Trans. Amer. Soc. Mech. Engrs* 80, 80-94 (1958).
- 9. G. W. SUTTON, Heat conduction relations for a subliming material during reentry. General Electric Co. Philadelphia, Pa., Report *TIS* 57 5*D*736 (October 1957).
- C. ECONOMOS, Ablation tests on plastic models in a hypersonic wind tunnel, J. Amer. Rocket Soc. 32, 1074-1081 (1962).
- L. STEG and H. LEW, Hypersonic ablation. Proceedings of AGARD Hypersonic Conference TCEA, Rhode-St. Genese, Belgium, April 3-6, 1962. Pergamon Press, New York (1964).
- M. C. ADAMS, Recent advances in ablation, J. Amer. Rocket Soc. 29, 625-632 (1959).
- 13. CHARLES D. HODGMAN, Handbook of Chemistry and Physics, 44 Edition. Chemical Rubber Publishing Co., Cleveland (1963).

Résumé—Une analyse approchée de la croissance et du champ de température intérieur d'un dépôt qui peut se produire sur une surface dans un écoulement gazeux, est étudiée en tenant compte du transport de chaleur par convection à la surface du dépôt. Des solutions simples pour un "temps court" et pour un "temps long" sont obtenues. Un exemple numérique est présenté qui peut avoir de l'intérêt en connection avec les surfaces cryogéniques dans une soufflerie hypersonique.

Zusammenfassung—Das Anwachsen und die Temperatur einer Ablagerung, die an einer kalten Oberfläche in einem Gasstrom auftreten kann, wird näherungsweise untersucht, wobei der konvektive Wärmeübergang an die Oberfläche der Ablagerung miteinbezogen wird. Man erhält einfache "Kurzzeit-" und "Langzeit-" -Lösungen. Ein Zahlenbeispiel, das in Verbindung mit sehr tiefen Oberflächentemperaturen in einem Hyperschallwindkanal interessieren dürfte, wird angegeben.

Аннотация— Разработан приближенный анализ процесса роста осадка (который может появляться на холодной поверхности в струе газа) и температуры внутри него при учете конвективного переноса тепла к поверхности осадка. Получены простые решения для «малого промежутка времени» и «большого промежутка времени». Приведен численный пример, который может представить интерес в связи с наличием криогенных поверхностей в сверхзвуковой аэродинамической трубе.